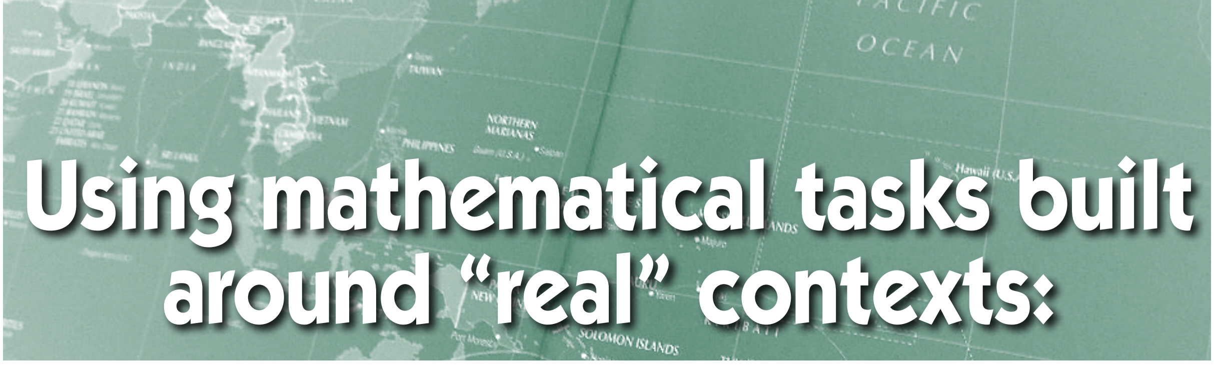

Using mathematical tasks built around "real" contexts:

Opportunities and challenges for teachers and students

The need for greater challenge and relevance for students in the middle years

DOUG CLARKE and **ANNE ROCHE,** describe how Type 2 tasks can be used to contextualise mathematics and result in positive outcomes for students.

From 1985 to 1989, Charles Lovitt and Doug Clarke coordinated the Mathematics Curriculum and Teaching Program (see, e.g., Lovitt & Clarke, 1988). During that time, they had many opportunities to ask teachers to identify their concerns about the teaching of mathematics in the middle years, which for the purposes of this article, we regard as Grades 5 to 8. Common responses can be summarised as follows: mathematics is seen by many students as boring and irrelevant; little thinking is involved; the subject is too abstract; there is a fear of failure; too much content is covered in too little depth; assessment is narrow; and it is a huge challenge to meet the needs of a wide range of abilities.

Recent reports indicate that these concerns and others remain issues in middle years' mathematics. For example, the Executive Summary of *Beyond the Middle* (Luke et al., 2003), a report commissioned by the Commonwealth Department of Education, Science and Training, and involving a literature review, a curriculum/policy mapping exercise, and system, school and classroom visits, included the following statement:

> There needs to be a more systematic emphasis on intellectual demand and student engagement in mainstream pedagogy… This will require a much stronger emphasis on quality and diversity of pedagogy, on the spread of mainstreaming

> of approaches to teaching and learning that stress higher order thinking and critical literacy, greater depth of knowledge and understanding and increases in overall intellectual demand and expectations of middle years students (p. 5).

The purpose of the Third International Mathematics and Science Study (TIMSS) Video Study was to investigate and describe mathematics and science teaching practices in a variety of countries. The researchers videotaped and analysed in great detail a total of 638 Grade 8 lessons from seven participating countries, including Australia. Altogether, 87 Australian schools and one teacher in each school were randomly selected in a way that was representative of all states, territories, school sectors, and metropolitan and country areas. Each teacher was filmed for one complete lesson.

The authors noted that Australian students would benefit from less repetitive work, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking. They noted that "there is an over-emphasis on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare." They noted "a syndrome of shallow teaching, where students are asked to follow procedures without reasons" (Hollingsworth, Lokan & McCrae, 2003, p. xxi). Although this study focussed on Year 8 classrooms, our experiences indicate that these descriptions could apply reasonably across Years 5 to 8.

As well as the documented problems in the kinds of mathematics offered to students and the ways in which it is presented, there is the affective domain. So many students look back on their experiences in the middle years mathematics with resentment, frustration, and an abiding belief that they "can't do mathematics."

So, there are many challenges facing teachers, schools and systems in improving both cognitive and affective aspects of students' mathematics learning in the middle years.

In the remainder of this article, we discuss the use by teachers of 'Type 2 tasks' within the Task Types in Mathematics Learning (TTML) Project, i.e., those in which the mathematics is situated within a contextualised practical problem. It is argued that these kinds of tasks have great potential for challenging and engaging students, and showing how mathematics can help us to make sense of the world.

We also discuss a major challenge, as we see it, for mathematics teachers generally, but particularly for those who take a problem solving approach to their teaching. The challenge is developing appropriate techniques and strategies needed in "pulling the lesson together."

What are Type 2 tasks?

When using Type 2 tasks as defined by the TTML Project, teachers situate mathematics within a contextualised practical problem where the motive is explicitly mathematics. This task type has a particular mathematical focus as the starting point and the context exemplifies this. The context serves the twin purposes of showing how mathematics is used to make sense of the world and motivating students to solve the task. For example, a Type 2 task can be created from the following question: "How many people can stand in your classroom?" (Lovitt & Clarke, 1988) where the task is of the kind, "Imagine we have the opportunity to put on a concert in this classroom with a local band to raise funds for more school computers. How many tickets should we sell?" Here, the context provides a motivation for what follows and dictates the mathematical decisions that the students make in finding a solution. The teacher will have broad intentions, in advance, about how the content relates to relevant curriculum documents, specifically, an understanding of area, estimation strategies, and the notion of

Figure 1. Signpost task.

measurement errors. Although the contexts are in some cases contrived (as with the lesson above), it is important to distinguish Type 2 tasks from word problems (e.g., Fennema, Franke, Carpenter & Carey, 1993), which are only contextualised in a very basic way. In typical word problems (e.g., I purchase a CD for $32.50; how much change would I receive from a $50 note?), the context is not intended to motivate students or help them to make sense of the world particularly, but largely provides what Maier (1991) called school problems coated with a thin veneer of "real world" associations.

Within the TTML Project, it is assumed that the teacher will pose the task, clarify terms, context and purpose, but will not tell the students what to do or how to do it. The teacher will orchestrate a class discussion after students have engaged with the task to hear interesting responses that teachers have specifically identified while the students are working, and will seek to draw out commonalities, and generalisations.

Where was this photo taken? An example of a Type 2 task

A number of teachers have used what we have come to call the Signpost task. The photos within this article and discussion are based largely on the use of the Signpost task developed by Doug Clarke and used by Anne Roche and Carli Kawalsky's in Carli's Year 5 class at Malvern Central School.

Setting the scene

The teacher asked students whether, during family travels, they had ever seen a sign at lookouts or at other tourist places which showed how far and in which direction a number of key places were from their current location. Some had seen such a sign (mostly at lookouts), and they shared their experiences. The teacher then held up a picture which had been taken of such a sign (shown in Figure 1) showing the distances in kilometres of 14 other cities from the signpost, and explained that today's lesson would involve the students working, in pairs, on trying to find out the location of the signpost.

Students were asked to offer initial thoughts on the location prior to setting them to work. The somewhat 'tropical' background can often be something of a distraction in their predictions.

The teacher then indicated that students were free to work on this problem in any way they wished. Atlases were provided for each pair.

Figure 2. Using an atlas.

Enabling prompts

Although most pairs decided upon a starting strategy and got to work, several students seemed unable to make a start on solving the problem and required some assistance. Sullivan, Mousley and Zevenbergen (2004) coined the term “enabling prompts” to refer to appropriate variations on the task or suggestions to students which might help those who are having trouble making a start on the problem. One helpful enabling prompt in this case was to suggest to students that they pick a city named on the sign and find out how far on the map it would be from the sign’s location and therefore which “mystery city” might contain this signpost.

This prompt seemed helpful, but it still provided a challenge, as it involved students using scale to see how the distances on the map related to the real distances in kilometres. For example, if Seoul is 9636 kilometres away from the signpost, what would this be in centimetres? Some students used the scale below by measuring with their ruler the length from zero to 4000, doubling this length to make 8000 km then adding the length from zero to round about 1600 to make a length that closely represented the distance to Seoul.

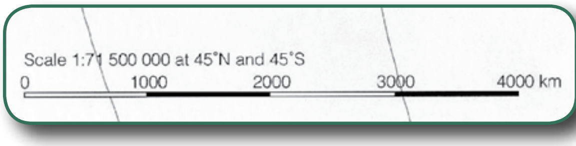

Figure 3. Scale on world map.

We acknowledge that scale is a challenging topic for Year 5 and not addressed until later years in some curriculum documents. However, the students in Carli’s class persisted in their problem solving attempts and appeared to learn a lot about scale along the way.

Having come up with an approximation, students then measured in a straight line that many centimetres in a variety of directions to establish possible locations for the signpost.

Students varied in their abilities to read and use a scale, and to convert the distances shown on the signpost to centimetres on the map. In discussion with teachers, it was clear

Figure 4. Making some calculations.

that students in the middle years rarely are asked to either create or use a scale, and that this topic probably needs greater attention.

It is important to stress here that because the map is a flat representation of the 3D world, all scales can only be used approximately. Nevertheless, most pairs were making a start, and starting to rule cities in and out as possibilities.

A more helpful strategy

For some students the enabling prompt was not necessary as they had noted that Sydney and Fiji were almost equidistant from the signpost (2159 km and 2157 km, respectively) and the closest places to the “mystery city.”

On one world map, the scale was expressed as “1 cm on the map represents 450 km on the ground.” Some students needed assistance to see that the calculation of 2159 ÷ 450 would give an approximate distance on the map from Sydney to the mystery city. Explaining that this equation is solving “How many lots of 450 would go into 2159?” seemed to be helpful. Once this was understood, they were encouraged to estimate this (“About 4 or 5,”) and then to find the answer using a calculator.

Some difficulties arose around the need to be accurate when converting and measuring, particularly when working with some of the

Figure 5. Locating a city.

larger distances. Students were free to choose any cities and atlas maps to work on, but not all of the signpost cities (nor the mystery city) were on the maps which students chose. This meant that students had to estimate or even guess where their cities might be. Students sometimes needed to move between maps of the world and maps of the Pacific region, once they realised that the mystery city was likely to be somewhere not that far from Australia.

Building upon students' insights

One pair of students had worked out that the mystery city was close to 5 cm from Fiji on the map. The pair then showed the teacher their insight that by rotating their ruler around Fiji, and noting where 5 cm from Fiji 'reached,' they knew that the city must be somewhere along this imaginary circle. The teacher took this opportunity to share the insight with the class, and introduce the use of a compass to make the measurements more accurate. It was not long before some students realised that drawing the same-sized circle around Sydney would provide other important information.

The challenge for students then was to take these two circles and decide what it actually told them. The first student to speak decided that it must be within the region created by the overlap of the two circles. The teacher pointed to a spot within the region, and asked the students whether this point was 5 cm from Fiji. After some further discussion, there was a consensus that the mystery city must be close to one of the two intersection points of the two circles.

The students then excitedly used other information about distances to establish that it must be the southernmost of the two points, leading to an answer of somewhere in New Zealand.

Figure 6. Using a compass to locate cities equidistant from Fiji.

Pulling it together

In most of the classes where this task was used, the teacher called upon a small number of pairs to share their reasoning. The pairs were generally chosen to represent a variety of different approaches and/or challenges faced. In several classes, teachers asked the students to talk about the mathematics they had learned (e.g., creating and using scales, careful measurement, using a compass to create circles, estimating, predicting and checking).

Some students also noted that strictly speaking, even two distances were enough to narrow down the possible cities to two (the intersection points).

Extensions

A number of key writers in the problem solving area (e.g., Brown & Walter, 1993) have stressed the importance of problem posing by students. Several teachers in the project took the opportunity in subsequent lessons to extend the work on the task, by encouraging students in groups to create their own signposts with cities of their own choice, and then to pose their problems to another group. This was an excellent way of consolidating the learning that had taken place during their work on the original task. Students appreciated being able to choose their own cities.

Project teachers' views on advantages and difficulties in using Type 2 tasks

Advantages: After at least one school term of trialling a range of Type 2 tasks, teachers were asked to comment on the "advantages of using this task type in your teaching." The comments below were typical:

- More hands on.
- Some were good for the student who struggles with mathematics.
- The mathematical skills and strategies are made purposeful and meaningful by being situated in a "real world" context.
- Increases the students' abilities to think.

- Allows the students to draw on a variety of understandings and topics — engaging and relevant to what they are doing.
- Engages advanced students. Combines knowledge and skills, e.g., a task may need measurement, calculation, logic.
- Each task can be taken in various directions by the students. There are different ways to solve the puzzle and are very engaging.

Difficulties: Teachers were asked, "What makes teaching this task type difficult?" Typical responses were the following:

- Some of the tasks were too challenging for support students [lower ability grouped students] and too long!
- The different learning needs and abilities of the students; at times some students arrived at their conclusions more quickly then others.
- Students who are less confident have very little idea of where to start if left to their own devices rather than assisted. These tasks can compound their negative feelings about themselves and maths.
- Not all the real situations are relevant to middle years' students and may not fit neatly into the existing curriculum.
- You need to do some preparation with the students. Students are more interested in the answer than the process.

Peter-Koop (2004) summarised many of the difficulties that students face when solving context-based problems, including comprehension of the text, and the identification of the mathematical core of the problem. Freudenthal (1984) referred to the construction of a *magical compatibility*, where an answer of 37.5 jeeps in a transport problem where students are asked to calculate the number of jeeps needed to transport soldiers, is seen by the students as perfectly acceptable.

It is worth noting that teachers in secondary schools generally found using the Type 2 tasks more challenging than did teachers in primary schools.

The challenge of pulling the lesson together: How do you "nail" the mathematics?

Most mathematics lessons can be considered to fall roughly into three phases:

(i) some kind of brief introduction to outline the proposed work for the day, to propose a problem or task, or to engage students with the mathematics through some motivational context;

(ii) an extended period of time when students work on the assigned problem(s) individually or in small groups; and

(iii) a whole class discussion where the teacher facilitates a conversation around the main mathematical points of the lesson.

One of the challenges teachers face when using Type 2 tasks is that of pulling the lesson together at the end (phase iii), in order to maximise the potential mathematics learning of students. Quite often, we observe mathematics classrooms where, for a variety of reasons (e.g., lack of time, the lesson heading into a detour which was unanticipated, class management issues, teacher confidence with the content), the lesson kind of just "dies." This means that the chance is not provided for the teacher to gain a clear sense of what students have learned from the activity, to make the mathematical focus clear for those students for whom it was not, to make connections with previous mathematics activity, to focus on where today's learning could be applied in other contexts, or to enable the students to learn from each other.

Alternatively, sometimes teachers encourage many students to share what they have found or what they have learned, but do not attempt to synthesise these comments, or they allow the conversation to remain at the trivial level, unrelated to the particular mathematical focus for the day (e.g., "I learned that I can do maths well if I try hard").

Although there are many possible ways of pulling a lesson together, we offer the following guidelines which may be helpful:

1. Being clear on the mathematical focus.

It may be stating the obvious, but it is important that the teacher begins the lesson having a clear idea of the mathematical focus of the day. In the case of the Signpost task, this might be that students will use scale to solve a practical problem. In clarifying the focus, a teacher might ask herself, “What is it that I want my students to know and be able to do after today’s lesson which they did not know and couldn’t do before the lesson?” In the case of a Type 2 task, this may include both a mathematical and contextual component. For the Signpost task, it therefore might include a greater awareness of the location of a number of cities around the world.

2. Considering the likely responses students will make to the tasks, and particularly the difficulties they might experience.

It is not always easy to anticipate how the lesson will “go,” but it is certainly worth thinking about the variety of ways in which students might respond to the task, so that these can be taken into account in pulling the lesson together. The teacher can be thinking about likely solution strategies, but also appropriate probing questions to ask during the main working part of the lesson and the pulling it together part.

3. Monitoring students’ responses to tasks as they work individually or in small groups on the tasks.

Although the teacher will have considered how students might respond to the task(s) in advance of the lesson (see part 2 above), things rarely go exactly according to plan, and it will be important for the teacher to observe the students at work and gain a sense of common strategies and difficulties.

One of the challenges in using any kind of task is maximising the chances that other students understand the solution paths offered by individuals during the “reporting time.” In order for the teacher to facilitate this well, it will be important for the teacher to understand the various strategies which are offered by students. A teacher will be well prepared for the discussion time if they have a clear sense, during individual or small group work, of the kinds of strategies which students are proposing and using.

4. Selecting students who will be invited to share during the discussion time.

In contrast to having everyone who wants to contribute having the “floor,” the monitoring above will enable the teacher to select carefully those students whose sharing will provide an opportunity to maximise the learning of the whole group. One possible ordering of reporting back is to have a student or group share first who made some progress but did not completely solve the problem, possibly revealing a common misconception or difficulty. This could be followed by a student or group who solved the problem in a satisfactory but common way. Finally, a student or group of students could present who provided an innovative and/or particularly elegant solution. It is likely that it will be sufficient for two or three students or groups to share.

5. Focusing on connections, generalisation and transfer.

Depending upon the task and the teacher’s purpose for it, the “pulling it together” phase provides the chance to encourage students to think about making connections between student solutions or connections with previous work. Another focus might be generalising from what they have learned and/or what can be transferred to new tasks, with questions like: “What kinds of things have we learned today which will help us to solve other problems?” “Could that method or strategy work no matter what numbers were involved?” “When would you use that particular strategy other than in tasks of this kind?”

Boaler (1993) provides an insight into the potential transfer of mathematical understanding when she notes that “it also seems likely that an activity which engages a student and enables her to attain some

personal meaning will enhance transfer to the extent that it allows deeper understanding of the mathematics involved” (p. 15). She notes that “school mathematics remains school mathematics for students when they are not encouraged to analyse mathematical situations and understand which aspects are central” (p. 17).

Watson and Mason (1988) provide a wonderful collection of prompts which encourage students to reflect on their learning, with a particular focus on generalising and specialising. For example, “What is the same and what is different about…?” “How can we be sure that…?” “What can change and what has to stay the same so that … is still true?” “Sort or organise the following according to…” “Tell me what is wrong with…”

In summary

In this article, we have described the features of Type 2 tasks as defined by the TTML Project, shown the way in which they have the potential to motivate students’ work in mathematics as they see how mathematics can help make sense of the world, and highlighted the kind of teacher actions which can ensure that the tasks are meaningful for students. We believe strongly that such tasks can support the development of connected mathematics learning for students in the middle years. As we have indicated, pulling the lesson together is a challenging but crucial skill in mathematics teaching. Hopefully, the discussion above provides some food for thought in maximising the learning which emerges from worthwhile tasks such as the Signpost one.

Where was the photograph taken?

The photograph was taken inside Auckland International Airport in New Zealand.

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